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Institution: Beijing-Dublin International College
Module: University Physics 2 (BDIC2008J)
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Problem Set 3

Electric Potential

Problem 1. *Suppose that in a lightning flash the potential difference between a cloud and the ground is* $1.0 \times 10^9\text{V}$ *and the quantity of charge transferred is* 30C*.* ***(a)*** *What is the change in energy of that transferred charge?* ***(b)*** *If all the energy released could be used to accelerate a* 1000kg *car from rest, what would be its final speed?*

Solution. **(a)** The change in energy of the transferred charge is

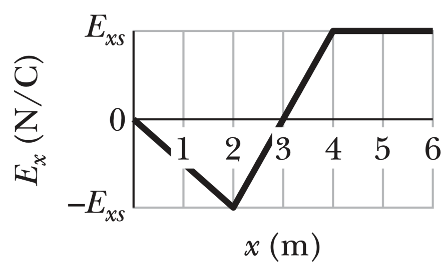
$$\Delta U = q\Delta V = (30\text{C})(1.0 \times 10^9\text{V}) = 3.0 \times 10^{10}\text{J}$$

(b) If all this energy is used to accelerate a 1000kg car from rest, then $\Delta U = E_k = mv^2/2$, and we find the car's final speed to be

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2\Delta U}{m}} = \sqrt{\frac{2(3.0 \times 10^{10}\text{J})}{1000\text{kg}}} = 7.7 \times 10^3\text{m/s}$$

□

Problem 2. *A graph of the x component of the electric field as a function of x in a region of space is shown in the figure. The scale of the vertical axis is set by $E_{xs} = 20.0\text{N/C}$. The y and z components of the electric field are zero in this region. If the electric potential at the origin is 10V, **(a)** what is the electric potential at $x = 2.0\text{m}$, **(b)** what is the greatest positive value of the electric potential for points on the x axis for which $0 \le x \le 6.0\text{m}$, and **(c)** for what value of x is the electric potential zero?*



Solution. **(a)** Recall the potential difference between two points is $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$. The change in potential is the negative of the "area" under the curve. Thus, using the area-of-a-triangle formula, we have

$$V - 10 = -\int_{x=0}^{x=2} \vec{E} \cdot d\vec{s} = \frac{1}{2}(2)(20) = 20$$

which yields $V = 30\text{V}$.

(b) For any region within $0 < x < 3\text{m}$, $-\int \vec{E} \cdot d\vec{s}$ is positive, but for any region for which $x > 3\text{m}$ it is negative. Therefore, $V = V_{max}$ occurs at $x = 3\text{m}$.

$$V - 10 = -\int_{x=0}^{x=3} \vec{E} \cdot d\vec{s} = \frac{1}{2}(3)(20) = 30$$

which yields $V_{max} = 40\text{V}$.

(c) In view of our result in part (b), we see that now (to find $V = 0$) we are looking for some $X > 3\text{m}$ such that the "area" from $x = 3\text{m}$ to $x = X$ is 40V. Using the formula for a triangle ($3 < x < 4$) and a rectangle ($4 < x < X$), we require

$$\frac{1}{2}(1)(20) + (X - 4)(20) = 40$$

Therefore, $X = 5.5\text{m}$. $\square$

Problem 3. *An infinite nonconducting sheet has a surface charge density* $\sigma = +5.80\text{pC/m}^2$. ***(a)*** *How much work is done by the electric field due to the sheet if a particle of charge* $q = +1.60 \times 10^{-19}\text{C}$ *is moved from the sheet to a point* P *at distance* $d = 3.56\text{cm}$ *from the sheet?* ***(b)*** *If the electric potential* V *is defined to be zero on the sheet, what is* V *at* P*?*

Solution. **(a)** The work done by the electric field is

$$W = \int_i^f q_0 \vec{E} \cdot d\vec{s} = \frac{q_0 \sigma}{2\varepsilon_0} \int_0^d dz = \frac{q_0 \sigma d}{2\varepsilon_0} = \frac{(1.60 \times 10^{-19}\text{C})(5.80 \times 10^{-12}\text{C/m}^2)(0.0356\text{m})}{2(8.85 \times 10^{-12}\text{C}^2/\text{N} \cdot \text{m}^2)}$$
$$= 1.87 \times 10^{-21}\text{J}$$

(b) Since

$$V - V_0 = -W/q_0 = -\sigma z / 2\varepsilon_0$$

with V_0 set to be zero on the sheet, the electric potential at P is

$$V = -\frac{\sigma z}{2\varepsilon_0} = -\frac{(5.80 \times 10^{-12}\text{C/m}^2)(0.0356\text{m})}{2(8.85 \times 10^{-12}\text{C}^2/\text{N} \cdot \text{m}^2)} = -1.17 \times 10^{-2}\text{V}$$

□

Problem 4. *A spherical drop of water carrying a charge of* 30pC *has a potential of* 500V *at its surface (with $V = 0$ at infinity). **(a)** What is the radius of the drop? **(b)** If two such drops of the same charge and radius combine to form a single spherical drop, what is the potential at the surface of the new drop?*

Solution. The electric potential for a spherically symmetric charge distribution falls off as $1/r$, where r is the radial distance from the centre of the charge distribution. The electric potential V at the surface of a drop of charge q and radius R is given by $V = q/4\pi\varepsilon_0 R$.

(a) With $V = 500\text{V}$ and $q = 30 \times 10^{-12}\text{C}$, we find the radius to be

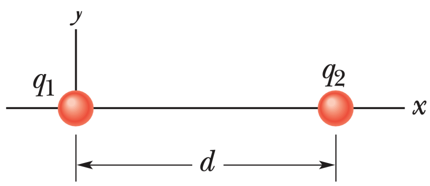
$$R = \frac{q}{4\pi\varepsilon_0 V} = \frac{(8.99 \times 10^9\text{N}\cdot\text{m}^2/\text{C}^2)(30 \times 10^{-12}\text{C})}{500\text{V}} = 5.4 \times 10^{-4}\text{m}$$

(b) After the two drops combine to form one big drop, the total volume is twice the volume of an original drop, so the radius R' of the combined drop is given by $4\pi(R')^3/3 = 2 * 4\pi R^3/3 \Rightarrow (R')^3 = 2R^3$ and $R' = 2^{1/3}R$. The charge then is twice the charge of the original drop: $q' = 2q$. Thus,

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{q'}{R'} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{2^{1/3}R} = 2^{2/3}V = 2^{2/3}(500\text{V}) \approx 790\text{V}$$

From this problem, we know a positively charged configuration produces a positive electric potential, and a negatively charged configuration produces a negative electric potential. Adding more charge increases the electric potential. □

Problem 5. *In the figure below, particles with the charges* $q_1 = +5e$ *and* $q_2 = -15e$ *are fixed in place with a separation of* $d = 24.0\text{cm}$. *With electric potential defined to be* $V = 0$ *at infinity, what are the finite* ***(a)*** *positive and* ***(b)*** *negative values of* x *at which the net electric potential on the* x *axis is zero?*



Solution. First, we observe that $V(x)$ cannot be equal to zero for $x > d$. In fact $V(x)$ is always negative for $x > d$. Now we consider the two remaining regions on the x axis: $x < 0$ and $0 < x < d$.

(a) For $0 < x < d$, we have $d_1 = x$ and $d_2 = d - x$. Let

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{x} + \frac{-3}{d-x}\right) = 0$$

and solve: $x = d/4$. With $d = 24.0\text{cm}$, we have $x = 6.00\text{cm}$.

(b) Similarly, for $x < 0$ the separation between q_1 and a point on the x axis whose coordinate is x is given by $d_1 = -x$; while the corresponding separation for q_2 is $d_2 = d - x$. We set

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{-x} + \frac{-3}{d-x}\right) = 0$$

to obtain $x = -d/2$. With $d = 24.0\text{cm}$, we have $x = -12.0\text{cm}$. $\square$

Problem 6. *The ammonia molecule* NH_3 *has a permanent electric dipole moment equal to* 1.47D*, where* $1\text{D} = 1$ *debye unit* $= 3.34 \times 10^{-30}\text{C} \cdot \text{m}$*. Calculate the electric potential due to an ammonia molecule at a point* 52.0nm *away along the axis of the dipole. (Set* $V = 0$ *at infinity.)*

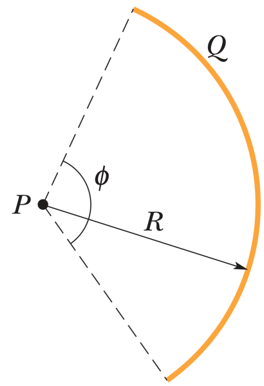
(Hint: The electric potential for electric dipole is $V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$*, in which* p *(*$= qd$*)) is the magnitude of the electric dipole moment* $\vec{p}$*. The vector* $\vec{p}$ *is directed along the dipole axis, from the negative to the positive charge. Thus,* θ *is measured from the direction of* $\vec{p}$*.)*

Solution. We use the Hint, where $\cos\theta = \cos 0^\circ = 1$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{(8.99 \times 10^9\text{N} \cdot \text{m}^2/\text{C}^2)(1.47 \times 3.34 \times 10^{-30}\text{C} \cdot \text{m})}{(52.0 \times 10^{-9}\text{m})^2} = 1.63 \times 10^{-5}\text{V}$$

□

Problem 7. *In the figure below, a plastic rod having a uniformly distributed charge* $Q = -25.6\text{pC}$ *has been bent into a circular arc of radius* $R = 3.71\text{cm}$ *and central angle* $\phi = 120°$*. With* $V = 0$ *at infinity, what is the electric potential at* P*, the centre of curvature of the rod?*



Solution. The potential is

$$V_P = \frac{1}{4\pi\varepsilon_0}\int_{rod}\frac{dq}{R} = \frac{1}{4\pi\varepsilon_0 R}\int_{rod} dq = \frac{-Q}{4\pi\varepsilon_0 R} = -\frac{(8.99\times 10^9\text{N}\cdot\text{m}^2/\text{C}^2)(25.6\times 10^{-12}\text{C})}{3.71\times 10^{-2}\text{m}}$$
$$= -6.20\text{V}$$

We note that the result is exactly what one would expect for a point-charge $-Q$ at a distance R. This "coincidence" is due, in part, to the fact that V is a scalar quantity. □

Problem 8. *What is the magnitude of the electric field at the point* $(3.00\vec{i} - 2.00\vec{j} + 4.00\vec{k})$m *if the electric potential in the region is given by* $V = 2.00xyz^2$*, where* V *is in volts and coordinates* x*,* y*, and* z *are in meters?*

Solution. The component of the electric field $\vec{E}$ in a given direction is the negative of the rate at which potential changes with distance in that direction. With $V = 2.00xyz^2$, we calculate the x, y, and z components of the electric field with partial derivative:

$$\begin{aligned} E_x &= -\frac{\partial V}{\partial x} = -2.00yz^2 \\ E_y &= -\frac{\partial V}{\partial y} = -2.00xz^2 \\ E_z &= -\frac{\partial V}{\partial z} = -4.00xyz \end{aligned}$$

which, at $(x, y, z) = (3.00\text{m}, -2.00\text{m}, 4.00\text{m})$, gives

$$(E_x, E_y, E_z) = (64.0\text{V/m}, -96.0\text{V/m}, 96.0\text{V/m})$$

The magnitude of the field is therefore

$$\begin{aligned} |\vec{E}| &= \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(64.0\text{V/m})^2 + (-96.0\text{V/m})^2 + (96.0\text{V/m})^2} \\ &= 150\text{V/m} = 150\text{N/C} \end{aligned}$$

□

Problem 9. *What is the "escape speed" for an electron initially at rest on the surface of a sphere with a radius of* 1.0cm *and a uniformly distributed charge of* $1.6 \times 10^{-15}\text{C}$*? That is, what initial speed must the electron have in order to reach an infinite distance from the sphere and have zero kinetic energy when it gets there? (Hint:* $m_e = 9.11 \times 10^{-31}\text{kg}$*,* $e = 1.60 \times 10^{-19}\text{C}$*.)*

Solution. The "*escape speed*" may be calculated from the requirement that the initial kinetic energy (of launch) be equal to the absolute value of the initial potential energy. Thus,

$$\frac{1}{2}mv^2 = \frac{eq}{4\pi\varepsilon_0 r}$$

where $m_e = 9.11 \times 10^{-31}\text{kg}$, $e = 1.60 \times 10^{-19}\text{C}$, $q = 10000e$, and $r = 0.010\text{m}$. This yields $v = 22490\text{m/s} \approx 2.2 \times 10^4\text{m/s}$. $\square$

Problem 10. *Two metal spheres, each of radius* 3.0cm*, have a center-to-center separation of* 2.0m*. Sphere 1 has charge* $+1.0 \times 10^{-8}$C*; sphere 2 has charge* -3.0×10^{-8}C*. Assume that the separation is large enough for us to say that the charge on each sphere is uniformly distributed (the spheres do not affect each other). With* $V = 0$ *at infinity, calculate* ***(a)*** *the potential at the point halfway between the centres and the potential on the surface of* ***(b)*** *sphere 1 and* ***(c)*** *sphere 2.*

Solution. The electric potential is the sum of the contributions of the individual spheres. Let q_1 be the charge on one, q_2 be the charge on the other, and d be their separation. The point halfway between them is the same distance $d/2(= 1.0\text{m})$ from the centre of each sphere.

For parts (b) and (c), we note that the distance from the centre of one sphere to the surface of the other is $d - R$, where R is the radius of either sphere. The potential of either one of the spheres is due to the charge on that sphere as well as the charge on the other sphere.

(a) The potential at the halfway point is

$$V = \frac{q_1 + q_2}{4\pi\varepsilon_0 d/2} = \frac{(8.99 \times 10^9\text{N} \cdot \text{m}^2/\text{C}^2)(1.0 \times 10^{-8}\text{C} - 3.0 \times 10^{-8}\text{C})}{1.0\text{m}} = -1.8 \times 10^2\text{V}$$

(b) The potential at the surface of sphere 1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R} + \frac{q_2}{d-R}\right] = (8.99{\times}10^9\text{N} \cdot \text{m}^2/\text{C}^2)\left[\frac{1.0 \times 10^{-8}\text{C}}{0.030\text{m}} - \frac{3.0 \times 10^{-8}\text{C}}{2.0\text{m} - 0.030\text{m}}\right] = 2.9 \times 10^3\text{V}$$

(c) Similarly, the potential at the surface of sphere 2 is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{d-R} + \frac{q_2}{R}\right] = (8.99{\times}10^9\text{N} \cdot \text{m}^2/\text{C}^2)\left[\frac{1.0 \times 10^{-8}\text{C}}{2.0\text{m} - 0.030\text{m}} - \frac{3.0 \times 10^{-8}\text{C}}{0.030\text{m}}\right] = -8.9 \times 10^3\text{V}$$

In the limit where $d \to \infty$, the spheres are isolated from each other and the electric potentials at the surface of each individual sphere become

$$V_{10} = \frac{q_1}{4\pi\varepsilon_0 R} = \frac{(8.99 \times 10^9\text{N} \cdot \text{m}^2/\text{C}^2)(1.0 \times 10^{-8}\text{C})}{0.030\text{m}} = 3.0 \times 10^3\text{V}$$

and

$$V_{20} = \frac{q_2}{4\pi\varepsilon_0 R} = \frac{(8.99 \times 10^9\text{N} \cdot \text{m}^2/\text{C}^2)(-3.0 \times 10^{-8}\text{C})}{0.030\text{m}} = -8.99 \times 10^3\text{V}$$

□